

TSI Assessment[©]

Mathematics and Statistics Standards



TEXAS COLLEGE AND CAREER
READINESS STANDARDS



TEXAS ESSENTIAL KNOWLEDGE
AND SKILLS



FIRST EDITION

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ELEMENTARY ALGEBRA and FUNCTIONS

§111.39. Algebra I, Adopted 2012 (One Credit).

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:
- (A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities;
 - (B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points;
 - (C) write linear equations in two variables given a table of values, a graph, and a verbal description;
 - (D) write and solve equations involving direct variation;
 - (H) write linear inequalities in two variables given a table of values, a graph, and a verbal description; and
 - (I) write systems of two linear equations given a table of values, a graph, and a verbal description.

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- (3) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:
- (A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$;
 - (B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems;
 - (C) graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems;
 - (D) graph the solution set of linear inequalities in two variables on the coordinate plane;
 - (E) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist;
 - (F) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems; and
 - (G) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.
- (4) Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:
- (B) compare and contrast association and causation in real-world problems; and
 - (C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.
- (5) Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:
- (A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides;
 - (B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides; and
 - (C) solve systems of two linear equations with two variables for mathematical and real-world problems.
- (8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:
- (A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.



§111.40. Algebra II, Adopted 2012 (One-Half to One Credit).

- (7) Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:
- (B) add, subtract, and multiply polynomials; and
 - (I) write the domain and range of a function in interval notation, inequalities, and set notation.
- (8) Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:
- (A) analyze data to select the appropriate model from among linear, quadratic, and exponential models.

§111.39. Algebra I, Adopted 2012 (One Credit). Continued

- (10) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:
- (A) add and subtract polynomials of degree one and degree two;
 - (B) multiply polynomials of degree one and degree two;
 - (C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend;
 - (D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property;
 - (E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two; and
 - (F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.
- (12) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:
- (A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function;
 - (B) evaluate functions, expressed in function notation, given one or more elements in their domains; and
 - (E) solve mathematic and scientific formulas, and other literal equations, for a specified variable.



INTERMEDIATE ALGEBRA AND FUNCTIONS

I. Numeric Reasoning

- A. Number representation
 - 1. Compare real numbers.
 - 2. Define and give example of complex numbers.
- B. Number operations
 - 1. Perform computations with real and complex numbers.

II. Algebraic Reasoning

- A. Expressions and equations
 - 1. Explain and differentiate between expressions and equations using words such as “solve,” “evaluate,” and “simplify.”
- B. Manipulating expressions
 - 1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- D. Representation
 - 1. Interpret multiple representations of equations and relationships.
 - 2. Translate among multiple representations of equations and relationships.

III. Geometric Reasoning

- A. Figures and their properties
 - 1. Identify and represent the features of plane and space figures.
 - 3. Recognize and apply right triangle relationships including basic trigonometry.
- B. Transformation and symmetry
 - 1. Identify and apply transformations to figures.
- C. Connection between geometry and other mathematical content strands
 - 1. Make connections between geometry and algebra.

IV. Measurement Reasoning

- C. Measurement involving geometry and algebra
 - 1. Find the perimeter and area of two-dimensional figures.
 - 3. Determine indirect measurements of figures using scale drawings, similar figures, the Pythagorean theorem, and basic trigonometry.

VI. Statistical Reasoning

- B. Describe data
 - 3. Compute and describe summary statistics of data.

VII. Functions

- B. Analysis of functions
 - 1. Understand and analyze features of a function.
- C. Model real world situations with functions
 - 1. Apply known function models.
 - 2. Develop a function to model a situation.



VIII. Problem Solving and Reasoning

A. Mathematical problem solving

3. Determine a solution.

C. Real world problem solving

1. Formulate a solution to a real world situation based on the solution to a mathematical problem.

IX. Communication and Representation

A. Language, terms, and symbols of mathematics

1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.



INTERMEDIATE ALGEBRA and FUNCTIONS

§111.39. Algebra I, Adopted 2012 (One Credit).

- (6) Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:
- (A) determine the domain and range of quadratic functions and represent the domain and range using inequalities; and
 - (C) write quadratic functions when given real solutions and graphs of their related equations.
- (7) Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:
- (A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x -intercept, y -intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry; and
 - (B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions.
- (8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:
- (A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.
- (9) Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:
- (A) determine the domain and range of exponential functions of the form $f(x) = abx$ and represent the domain and range using inequalities;
 - (B) interpret the meaning of the values of a and b in exponential functions of the form $f(x) = abx$ in real-world problems;
 - (C) write exponential functions in the form $f(x) = abx$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay; and

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- (D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems

§111.40. Algebra II, Adopted 2012 (One-Half to One Credit).

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Attributes of functions and their inverses. The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to:
- (A) graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = 3\sqrt{x}$, $f(x) = bx$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e , and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval;
- (3) Systems of equations and inequalities. The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve, and analyze reasonableness of solutions. The student is expected to:
- (C) solve, algebraically, systems of two equations in two variables consisting of a linear equation and a quadratic equation;
 - (D) determine the reasonableness of solutions to systems of a linear equation and a quadratic equation in two variables;
 - (E) formulate systems of at least two linear inequalities in two variables;
 - (F) solve systems of two or more linear inequalities in two variables; and
 - (G) determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.



- (4) Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:
- (A) write the quadratic function given three specified points in the plane;
 - (B) write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening;
 - (D) transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ to identify the different attributes of $f(x)$;
 - (F) solve quadratic and square root equations; and
 - (G) identify extraneous solutions of square root equations.
- (5) Exponential and logarithmic functions and equations. The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:
- (A) determine the effects on the key attributes on the graphs of $f(x) = bx$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative real values of a , c , and d ;
 - (B) formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation; and
 - (D) solve exponential equations of the form $y = abx$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions.
- (6) Cubic, cube root, absolute value and rational functions, equations, and inequalities. The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:
- (B) solve cube root equations that have real roots;
 - (D) formulate absolute value linear equations;
 - (E) solve absolute value linear equations;
 - (F) solve absolute value linear inequalities;
 - (H) formulate rational equations that model real-world situations;
 - (I) solve rational equations that have real solutions;



§111.40. Algebra II, Adopted 2012 (One-Half to One Credit).

- (J) determine the reasonableness of a solution to a rational equation;
 - (K) determine the asymptotic restrictions on the domain of a rational function and represent domain and range using interval notation, inequalities, and set notation; and
 - (L) formulate and solve equations involving inverse variation.
- (7) Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:
- (A) add, subtract, and multiply complex numbers;
 - (B) add, subtract, and multiply polynomials;
 - (C) determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two;
 - (D) determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods;
 - (E) determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping;
 - (F) determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two;
 - (G) rewrite radical expressions that contain variables to equivalent forms;
 - (H) solve equations involving rational exponents; and
 - (I) write the domain and range of a function in interval notation, inequalities, and set notation.
- (8) Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:
- (A) analyze data to select the appropriate model from among linear, quadratic, and exponential models; and
 - (C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.



GEOMETRY AND MEASUREMENT

I. Numeric Reasoning

- B. Number operations
 - 1. Perform computations with real and complex numbers.

II. Algebraic Reasoning

- B. Manipulating expressions
 - 1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- C. Solving equations, inequalities, and systems of equations
 - 1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to solve equations, inequalities, and systems of linear equations.
- D. Representation
 - 1. Interpret multiple representations of equations and relationships.
 - 2. Translate among multiple representations of equations and relationships.

III. Geometric Reasoning

- A. Figures and their properties
 - 3. Recognize and apply right triangle relationships including basic trigonometry.
- B. Transformation and symmetry
 - 1. Identify and apply transformations to figures.
- C. Connection between geometry and other mathematical content strands
 - 1. Make connections between geometry and algebra.

IV. Measurement Reasoning

- A. Measurement involving physical and natural attributes
 - 1. Select or use the appropriate type of unit for the attribute being measured.
- B. Systems of measurement
 - 2. Convert within a single measurement system.
- C. Measurement involving geometry and algebra
 - 1. Find the perimeter and area of two-dimensional figures.
 - 2. Determine the surface area and volume of three-dimensional figures.
 - 3. Determine indirect measurements of figures using scale drawings, similar figures, the Pythagorean theorem, and basic trigonometry.

VII. Functions

- B. Analysis of functions
 - 1. Understand and analyze features of a function.
- C. Model real world situations with functions
 - 1. Apply known function models.

VIII. Problem Solving and Reasoning

- C. Real world problem solving
 - 1. Formulate a solution to a real world situation based on the solution to a mathematical problem.

IX. Communication and Representation

- A. Language, terms, and symbols of mathematics
 - 1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
 - 2. Use mathematical language to represent and communicate the mathematical concepts in a problem.



GEOMETRY and MEASUREMENT

§111.41. Geometry, Adopted 2012 (One Credit).

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:
- (A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint; and
 - (B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines.
- (3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:
- (A) describe and perform transformations of figures in a plane using coordinate notation;
 - (B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;
 - (C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and
 - (D) identify and distinguish between reflectional and rotational symmetry in a plane figure.



§111.41. Geometry, Adopted 2012 (One Credit). Continued

- (4) Logical argument and constructions. The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to:
- (A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems; and
 - (D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.
- (5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:
- (A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools; and
 - (D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.
- (6) Proof and congruence. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:
- (A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems;
 - (C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;
 - (D) verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems; and
 - (E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems.
- (7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:
- (A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and
 - (B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.



§111.41. Geometry, Adopted 2012 (One Credit). Continued

- (8) Similarity, proof, and trigonometry. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:
- (A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems.
- (10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:
- (A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and
 - (B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.
- (11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:
- (A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure;
 - (B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;
 - (C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and
 - (D) apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.
- (12) Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:
- (A) apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;
 - (B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems; and
 - (C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems.



DATA ANALYSIS, PROBABILITY, AND STATISTICS

I. Numeric Reasoning

- A. Number representation
 - 1. Compare real numbers.
- B. Number operations
 - 1. Perform computations with real and complex numbers.
- C. Number sense and number concepts
 - 1. Use estimation to check for errors and reasonableness of solutions.

II. Algebraic Reasoning

- B. Manipulating expressions
 - 1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
- C. Solving equations, inequalities, and systems of equations
 - 1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to solve equations, inequalities, and systems of linear equations.
- D. Representation
 - 1. Interpret multiple representations of equations and relationships.

III. Geometric Reasoning

- A. Figures and their properties
 - 1. Identify and represent the features of plane and space figures.
 - 2. Make, test, and use conjectures about one-, two-, and three-dimensional figures and their properties.
 - 3. Recognize and apply right triangle relationships including basic trigonometry.
- B. Transformation and symmetry
 - 1. Identify and apply transformations to figures.
 - 2. Identify the symmetries of a plane figure.
- C. Connection between geometry and other mathematical content strands
 - 1. Make connections between geometry and algebra.
 - 3. Make connections between geometry and measurement.

IV. Measurement Reasoning

- A. Measurement involving physical and natural attributes
 - 1. Select or use the appropriate type of unit for the attribute being measured.
- B. Systems of measurement
 - 1. Convert from one measurement system to another.
 - 2. Convert within a single measurement system.
- C. Measurement involving geometry and algebra
 - 1. Find the perimeter and area of two-dimensional figures.
 - 2. Determine the surface area and volume of three-dimensional figures.
 - 3. Determine indirect measurements of figures using scale drawings, similar figures, the Pythagorean theorem, and basic trigonometry.



VII. Functions

C. Model real world situations with functions

1. Apply known function models.

VIII. Problem Solving and Reasoning

A. Mathematical problem solving

1. Analyze given information.
3. Determine a solution.

C. Real world problem solving

1. Formulate a solution to a real world situation based on the solution to a mathematical problem.

IX. Communication and Representation

A. Language, terms, and symbols of mathematics

1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.



DATA ANALYSIS, PROBABILITY, AND STATISTICS

§111.41. Geometry, Adopted 2012 (One Credit).

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (13) Probability. The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to:
- (A) develop strategies to use permutations and combinations to solve contextual problems;
 - (B) determine probabilities based on area to solve contextual problems;
 - (C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;
 - (D) apply conditional probability in contextual problems; and
 - (E) apply independence in contextual problems.

